POLYNOMIALS

1. Which of the following is not a polynomial?
   (a) $x^2+\sqrt{2}x+3$ (b) $x^2+\sqrt{2x}+6$ (c) x^3+3x^2-3 (d) $6x+4$

2. The degree of the polynomial $3x^3 - x^4 + 5x + 3$ is
   (a) –4 (b) 4 (c) 1 (d) 3

3. Zero of the polynomial $p(x) = a^2x$, $a \neq 0$ is
   (a) $x = 0$ (b) $x = 1$ (c) $x = -1$ (d) $a = 0$

4. Which of the following is a term of a polynomial?
   (a) 2x (b) $\frac{3}{x}$ (c) $x^{\sqrt{x}}$ (d) $\sqrt{x}$

5. If $p(x) = 5x^2 - 3x + 7$, then $p(1)$ equals
   (a) –10 (b) 9 (c) –9 (d) 10

6. Factorisation of $x^3 + 1$ is
   (a) $(x + 1)(x^2 - x + 1)$ (b) $(x + 1)(x^2 + x + 1)$
   (c) $(x + 1)(x^2 - x - 1)$ (d) $(x + 1)(x^2 + 1)$

7. If $x + y + 2 = 0$, then $x^3 + y^3 + 8$ equals
   (a) $(x + y + 2)^3$ (b) 0 (c) 6xy (d) –6xy

8. If $x = 2$ is a zero of the polynomial $2x^2 + 3x - p$, then the value of p is
   (a) –4 (b) 0 (c) 8 (d) 14

9. $x+\frac{1}{x}$ is
   (a) a polynomial of degree 1 (b) a polynomial of degree 2
   (c) a polynomial of degree 3 (d) not a polynomial

10. Integral zeroes of the polynomial $(x + 3)(x - 7)$ are
    (a) –3, –7 (b) 3, 7 (c) –3, 7 (d) 3, –7

11. The remainder when $p(x) = 2x^2 - x - 6$ is divided by $(x - 2)$ is
    (a) $p(-2)$ (b) $p(2)$ (c) $p(3)$ (d) $p(-3)$

12. If $2\left(a^2+b^2\right)=\left(a+b\right)^2$, then
    (a) $a + b = 0$ (b) $a = b$ (c) $2a = b$ (d) $ab = 0$

13. If $x^3 + 3x^2 + 3x + 1$ is divided by $(x + 1)$, then the remainder is
    (a) –8 (b) 0 (c) 8 (d) $\frac{1}{8}$

14. The value of $(525)^2 - (475)^2$ is
    (a) 100 (b) 1000 (c) 100000 (d) –100

15. If $a + b = -1$, then the value of $a^3 + b^3 - 3ab$ is

(a) –1 (b) 1 (c) 26 (d) –26

16. The value of $(2-a)^3 + (2-b)^3 + (2-c)^3 - 3(2-a)(2-b)(2-c)$ when $a + b + c = 6$ is

(a) –3 (b) 3 (c) 0 (d) –1

17. If $\frac{a}{b} + \frac{b}{a} = 1, (a \neq 0, b \neq 0)$, then the value of $a^3 - b^3$ is

(a) –1 (b) 0 (c) 1 (d) $\frac{1}{2}$

18. If $x = \frac{1}{2-\sqrt{3}}$, then the value of $(x^2 - 4x + 1)$ is

(a) –1 (b) 0 (c) 1 (d) 3

19. The number of zeroes of the polynomial $x^3 + x - 3 - 3x^2$ is

(a) 1 (b) 2 (c) 0 (d) 3

20. If $(x + 2)$ and $(x - 2)$ are factors of $ax^4 + 2x - 3x^2 + bx - 4$, then the value of $a + b$ is

(a) –7 (b) 7 (c) 14 (d) –8

POLYNOMIALS

1. Factorize the following: $9x^2 + 6x + 1 - 25y^2$.

2. Factorize the following: $a^2 + b^2 + 2ab + 2bc + 2ca$

3. Show that $p(x) = x^3 - 3x^2 + 2x - 6$ has only one real zero.

4. Find the value of a if $x + 6$ is a factor of $x^3 + 3x^2 + 4x + a$.

5. If polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ leaves the same remainder when each is divided by $x - 4$, find the value of a..

6. The polynomial $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ when divided by $(x - 1)$ and $(x + 1)$ leaves the remainders 5 and 19 respectively. Find the values of a and b. Hence, find the remainder when $f(x)$ is divided by $(x - 2)$.

7. If the polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 2x + a$ leave the same remainder when divided by $(x - 2)$, find the value of a. Also, find the remainder in each case.

8. If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a.

9. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$ leaves the remainder 19. Find the values of a. Also find the remainder when $p(x)$ is divided by $x + 2$.

10. If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that $p = r$.

11. Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.

12. Simplify $(2x - 5y)^3 - (2x + 5y)^3$.

13. Multiply $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $(-z + x - 2y)$.

14. If a, b, c are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$

15. If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

16. Without actual division, prove that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

17. Without actual division, prove that $x^3 - 3x^2 - 13x + 15$ is exactly divisible by $x^2 + 2x - 3$.

18. Find the values of a and b so that the polynomial $x^3 - 10x^2 + ax + b$ is exactly divisible by $(x - 1)$ as well as $(x - 2)$.

19. Find the integral zeroes of the polynomial $2x^3 + 5x^2 - 5x - 2$.

20. If $(x - 3)$ and $\left(x - \frac{1}{3}\right)$ are both factors of $ax^2 + 5x + b$, then show that $a = b$.

21. Find the values of a and b so that the polynomial $x^4 + ax^3 - 7x^2 + 8x + b$ is exactly divisible by $(x + 2)$ as well as $(x + 3)$.

22. If $x^3 + ax^2 + bx + 6$ has $(x - 2)$ as a factor and leaves a remainder 3 when divided by $(x - 3)$, find the values of a and b.

23. Find the value of $x^3 + y^3 + 15xy - 125$ if $x + y = 5$.

24. Without actually calculating, find the value of $(25)^3 - (75)^3 + (50)^3$.

25. Factorise each of the following cubic expressions:

(i) $8x^3 - y^3 - 12x^2y + 6xy^2$

(ii) $27q^3 - 125p^3 - 135q^2p + 225qp^2$

(iii) $8x^3 + 729 + 108x^2 + 486x$

(iv) $27x^3 - \frac{1}{216} - \frac{9}{2}x^2 + \frac{1}{4}x$

26. Factorise:

(i) $x^3 + 216y^3 + 8z^3 - 36xyz$

(ii) $a^3 - 64b^3 - 27c^3 - 36abc$

27. Factorise: $\left(\frac{1}{2}x - 3y\right)^3 + \left(3y - \sqrt{3}z\right)^3 + \left(\sqrt{3}z - \frac{1}{2}x\right)^3$

28. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

29. Find a zero of the polynomial $p(x) = 2x + 1$.

30. Verify whether 2 and 0 are zeroes of the polynomial $x^2 - 2x$.

31. Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$ (ii) $p(x) = x - 5$ (iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$ (v) $p(x) = 3x$ (vi) $p(x) = ax$, $a \neq 0$

32. Find the value of each of the following polynomials at the indicated value of variables:

(i) $p(x) = 5x^2 - 3x + 7$ at $x = 1$.

(ii) $q(y) = 3y^3 - 4y + \sqrt{11}$ at $y = 2$.

(iii) $p(t) = 4t^4 + 5t^3 - t^2 + 6$ at $t = a$.

33. Divide $p(x)$ by $g(x)$, where $p(x) = x + 3x^2 - 1$ and $g(x) = 1 + x$.

34. Divide the polynomial $3x^4 - 4x^3 - 3x - 1$ by $x - 1$.

35. Find the remainder obtained on dividing $p(x) = x^3 + 1$ by $x + 1$.

36. Find the remainder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$.

37. Check whether the polynomial $q(t) = 4t^3 + 4t^2 - t - 1$ is a multiple of $2t + 1$.

38. Check whether p(x) is a multiple of g(x) or not, where $p(x) = x^3 - x + 1$, $g(x) = 2 - 3x$.

39. Check whether g(x) is a factor of p(x) or not, where $p(x) = 8x^3 - 6x^2 - 4x + 3$, $g(x) = \frac{x}{3} - \frac{1}{4}$.

40. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

41. Examine whether $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$ and of $2x + 4$.